LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2013**

MT 3812 - CLASSICAL MECHANICS

Date : 07/11/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

Answer ***ALL*** the questions

01. a. i. Classical dynamics restricted to those systems of interacting bodies for which quantum mechanical effects are ----

ii. Nature is the work by ---- that people think of when they think of physics.

iii. Newton invented the branch of mathematics ------ in order to solve difficult mathematical problems

iv. ----- is an abstract physical quantity that is not easily perceived by humans.

v. The law of conservation of energy was first hypothesized by ----------

OR

b. State and prove the principle of virtual work

[5]

c. Derive the Lagrange's equation of motion and find the differential equation of motion of a simple pendulum of length l.

OR

d. State and prove the Variational principle and Prove that the shortest distance between two points in a plane is a straight line.

[15]

02. a. i.Generalized momenta: p_k =

ii. Relation between the Lagrangian L and the Hamiltonian H is ------

iii.For a simple dynamical system, H = KE + ----

iv. Hamilton's canonical equations of motion: $\frac{\partial H}{\partial q_k}$ = ---- and $\frac{\partial H}{\partial p_k}$ = ------

v. An ignorable coordinate is one which is absent from --------

OR

b. Find the Routh's function for the motion of a particle in the central force field.

[5]

c. i. State Hamilton's principle and deduce Lagrange's equation from Hamilton's principle. And hence find the equation of one dimension Harmonic oscillator.

[7+8]

OR

d. State and prove Hamilton's principle of least action

[15]

03. a. i. If both q, p are periodic function of time with same frequency then the periodic motion is known as-------

ii. ---------has both kinds of periodicity.

iii. The angles θ, ϕ, ψ are known as -------

iv. If the dynamical system have one degree freedom, then pdq – PdQ = ----------

v. $G_2 = \sum_j q_j P_j$ generates an ---------- transformation.

OR

b. Show that $Q = (2q)^{1/2} e^k \cos p, P = (2q)^{1/2} e^{-k} \sin p$ is a canonical transform. **[5]**

c. Discuss about the motion of a top by using
i. Lagrange's method
ii. Hamilton's method
[**7+8**]

OR

d. State and prove Integral invariant theorem of Point care
[**15**]

04. a. i. The solution of $H(q_1, q_2 q_{3,\ldots} q_n, \frac{\partial F_2}{\partial q_1}, \frac{\partial F_2}{\partial q_2}, \frac{\partial F_2}{\partial q_3}, \ldots \frac{\partial F_2}{\partial q_n}, t) + \frac{\partial F_2}{\partial t} = 0$ is known as ----

ii. If u is a function of q_i, p_i, t then $\frac{du}{dt} = [u, H] +$ ---

iii. $v[u,w] + w[u,v] = ---$

iv. If q_i is cyclic, then p_i is a -----

v. $\sum_{l=1}^{2n} \{u_l, u_i\}[u_l, u_j] = ----$

OR

b. Derive the transformation equation for infinite decimal contact transformation in terms of Poisson bracket **[5]**

c. Derive the conservation theorem of angular momentum using infinite decimal contact transformation

OR

d. State and prove Jacobi's Identity **[15]**

05. a. i. The *Complete* integral W of Hamilton –Jacobi equation $H(q_k, \frac{\partial W'}{\partial q_k}) = \alpha_1$ is called -------

ii. Separation of variables in Hamilton Jacobi's equation is possible only if ------

iii. For a conservative dynamical system in which the generalized coordinates θ , ϕ are cyclic and r is noncyclic , then the solution is given by ----------

vi. If the variables q_k , k = 1,…n are cyclic, then the solution is of the form ----------

v. If W_k denotes characteristic function, then $J_k = \int \frac{\partial W_k}{\partial q_k} dq_k$ is known as ----------

OR

b. Discuss the motion of a particle moving in a plane under the action of central **[5]**

c. Discuss the Harmonic Oscillator problem using Hamilton Jacobi equation

OR

d. Discuss Kepler's problem using action angle variable. [15]